# Density forecasts of inflation: a quantile regression forest approach

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# Density Forecasting

- Adequate policy response requires to assess the temporary/persistent nature of past, current (and future, to a certain extent) "facts" for inflation dynamics
- The future is uncertain: economic policy is based on an analysis of the likelihood of different events (risk analysis)

$\Rightarrow$ **Accurate density forecasts are a fundamental input for monetary policy**

# Inflation dynamics

- The Eurosystem analysis of inflation dynamics is heavily skewed toward linear models (Darracq-Paries et al. 2021)
- Yet, non-linearities are often argued to play an important role for inflation dynamics
  - Long and ongoing debate about steepening/flattening of the Phillips Curve (e.g. Del Negro et al. 2020; Eser et al. 2020; Costain et al. 2022)
  - Evidence that some "puzzling" inflation dynamics may be reconciled with theory by invoking non-linearities (Linde and Trabandt, 2019; Forbes et al. 2021)
- However, so far non-linearity has not dominated the landscape of modelling in support of monetary policy

⇒ **How relevant it is for inflation density forecasting to account for non-linearity? And which non-linearity?**

# Aim of this paper

- Design and evaluate the accuracy of a new model for euro area density inflation forecasting
- No commitment to one type of non-linearity
- Assess the role of non-linearities for euro area (headline and core) inflation dynamics, by controlling for "overfitting" (out-of-sample accuracy criterion)

**⇒ Quantile regression forests (a variant of Random Forests) as a way to operationalize non-parametric models**

# Empirical strategy

- Define our measure of prices as $p_t$. Assume we have data until time (i.e. month) $t$. $h = (3, 6, 9, 12)$ months:

$$\pi_t^h = (1200/h) \times [p_t/p_{t-h} - 1]$$

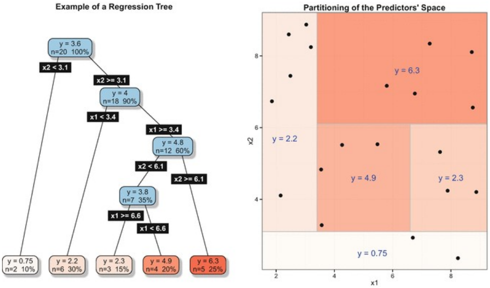
- Estimate $\pi_t = m(\pi_{t-h} \ldots \pi_{t-h-p}; x_{t-h} \ldots x_{t-h-k}) + \varepsilon_t$

- Project forward: $\hat{\pi}_{t+h} = m(\pi_t \ldots \pi_{t-p}; x_t \ldots x_{t-k})$

**Main ingredients**

- Direct density forecast
- m(.) quantile regression forest (variant of the random forest)

# Regression trees



# Non-linearity: Regression trees

- Regression trees allow very general relationships between predictors and the target variable
- However, regression trees are normally bad forecasting models, high variance, overfitting
  - One could "prune" them (akin to shrinkage), reducing ex ante their ability to (over-)fit
  - Normally, not the path taken in the literature

**Variance reduction is rather achieved by combination of several trees: random forests**

# The idea of Random Forests - Breiman 2001

1. **Bootstrap** observations (and keep the "out-of bag" observations)
2. Grow **many** trees
3. In each tree, use only a (randomly chosen) **sub-set of predictors** at each node
4. **Combine** the predictions of the trees at the end

# Does this make sense?

- Combination reduces variance of the forecasts
- Variance reduction maximized when the predictions are not correlated
    - Bootstrap to ensure "diversity" in the trees
    - The randomization step further de-correlates the trees
- Density forecasts: rather than taking averages of the target variable in the last nodes, compute sample quantiles $\Rightarrow$ **Quantile Regression Forest** (Meinshausen, 2006)
- One issue with regression trees/random forests: they do not extrapolate. How long can it take to adapt to unprecedented developments? Is "conservativeness" good or bad?

# Out-of-sample accuracy

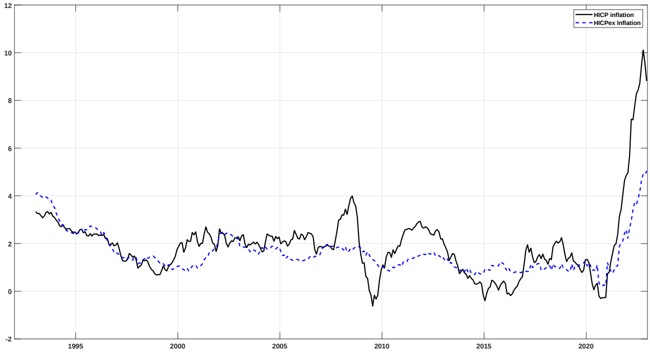
- Full sample: January 1992 - December 2022
- About twenty years of out-of-sample evaluation (first estimation sample until end 2001)
- Update by one of observation and re-estimate the model (recursive scheme)
- Forecast horizon: 3, 6, 9 and 12 months ahead; 20 years of out-of-sample evaluation
- CRPS for density forecasts. RMSE for point forecasts

# Benchmark models

- Combination of 500 5-variate B-VARs (randomly drawn regressors) - VARCOMB
    - Ensemble of linear models to isolate as much as possible the linear/non-linear dimension
- Comparison with survey and institutional forecasts (SPF and BMPE, today only focus on BMPE)
    - Compare with judgemental/very sophisticated forecasts - is QRF useful for a policy instution?

# Targets - headline and core inflation

Figure: Headline and Core Inflation - year-on-year




Note: Headline inflation: black solid line; Core inflation: blue dashed line.

# Out-of-sample accuracy - summary

- The QRF is competitive with the linear benchmark
  - Slightly better for core inflation (by a small margin);
  - On par for headline inflation at short horizons and worse for long horizons
- Also competitive with institutional forecasts, especially at short horizons

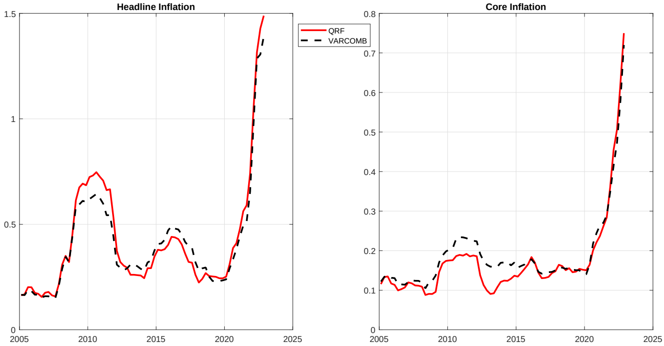
**⇒ QRF more a complement than a substitute for traditional technques**

**⇒ Non-linearity not that pervasive. Direct effects of commodity prices "linear", non-linearity is more evident for core inflation**

# Focus on some results

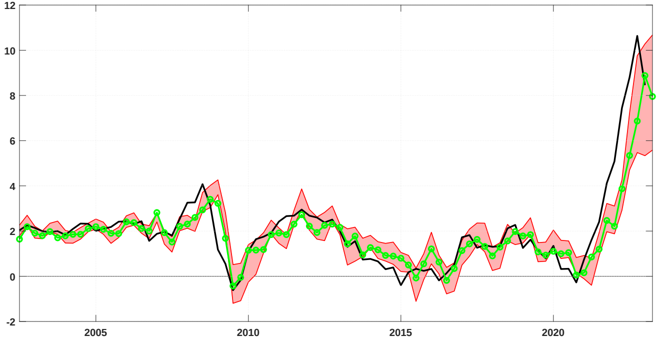
- Performance over time: similar (on average) but different (over time)
- Judgement and non-linearity
- Opening the black-box: contributions of different predictors

# Comparison with linear model, h=6



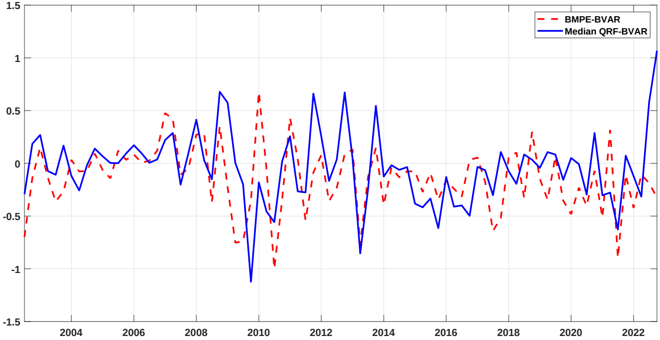

Note: Red solid line: QRF; Black dashed line: VARCOMB. The value on the vertical axis at each point refers to the average CRPS over the current quarter and the previous eleven quarters.

# Headline Inflation, QRF and BMPE point forecast, h=6



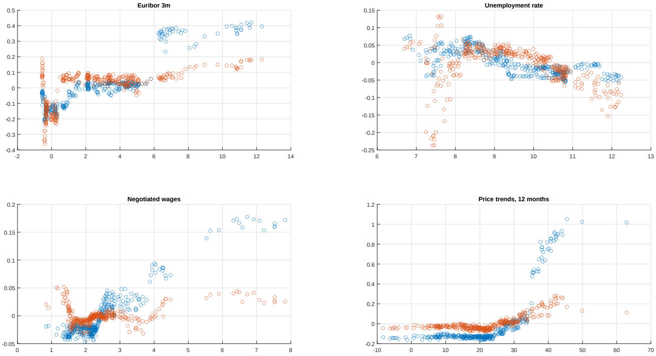

Note: Black solid line: year-on-year growth rate of HICP; Red area: 16th-84th quantiles QRF density forecasts; Green line with circles: BMPE projections.

# Distance from non-linearity: gaps of BMPE and QRF versus VARCOMB




Note: Solid blue line: six months ahead (median) QRF forecast of headline inflation minus corresponding VARCOMB forecast; Dashed red line: six months ahead BMPE forecast of headline inflation minus corresponding VARCOMB forecast.

# Non-linearities: Shapley Values - Top Contributors




Note: Vertical axis: in-sample Shapley values for the variable indicated in the title for headline inflation (red) and core inflation (blue). Horizontal axis: value of the variable indicated in the title

# Conclusion

- The quantile regression forest (QRF) is a welcome addition to the Eurosystem toolbox to forecast inflation. Complement rather than substitute the currently available tools
- Non-linearity in inflation dynamics: perhaps, mostly for core inflation
- QRF quite similar to BMPE/SPF, both in dynamics and accuracy - judgement partly adds "non-linearity"?

# BACKGROUND SLIDES

# What we find - comparison with state-of-the-art linear models and judgemental institutional

- The quantile regression forest (QRF) is a good forecasting model, especially at short horizons and for core inflation
- Overall, similar accuracy with state-of-the-art linear models on full sample. Different accuracy in sub-samples, diversity in the toolbox

⇒ **Complementarity of the approaches. Non-linearity maybe more relevant in specific episodes and for core inflation.**

- QRF is good in terms of relative accuracy, despite not being able to incorporate future info using judgement
- Quite strong collinearity with (judgemental) Eurosystem forecasts!

⇒ **Judgement may be adding mild non-linearity to the Eurosystem forecasts.**

# Related literature

**Inflation forecasting**

Large literature, see Faust and Wright (2013) and my forthcoming survey with M. Banbura and J. Paredes. To be singled out: Medeiros et al. (2021), Random Forest for point US inflation forecast

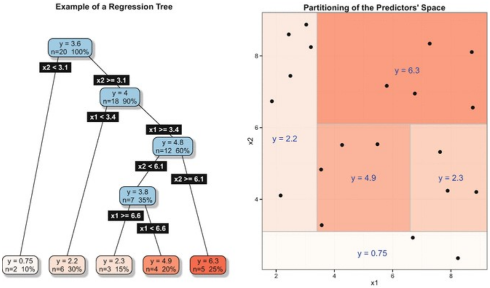
**Non-linearity in inflation dynamics**

See, for example Akerlof et al., 1996; Costain et al., 2022; Fahr and Smets, 2010; Benigno and Ricci, 2011; Linde and Trabandt, 2019; Del Negro et al. 2020; Forbes et al., 2021; Goulet-Coulombe et al., 2022

**Ensemble methods for prediction**

See, for example, Athey et al., 2019; Avramov, 2002; Bai and Ng, 2009; Cremers, 2002; Faust et al., 2013; Fernandez et al., 2001; Inoue and Kilian, 2008; Jin et al., 2014; Ng, 2013; Rapach and Strauss, 2010; Sala-I-Martin et al., 2004; Varian, 2014; Wager and Athey, 2018; Wright, 2009; Giannone et al., 2021; Medeiros et al., 2021; Clark et al. 2022a; Clark et al. 2022b

# Non-linearity: Regression trees



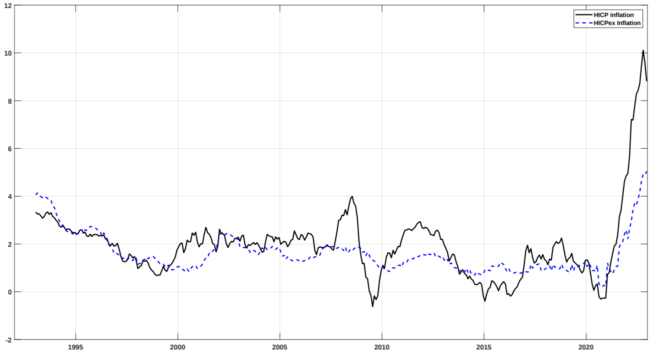
# Hyperparameters

The specification choices for Random Forests are just a few. In general, we take the default choices in the literature (but further tuning is possible)

- **Depth of trees**
  - Control overfitting
  - We experimented with several configurations (varying number of splits, number of observations in the last node) - no impact on results
- **Number of variables randomly drawn for each split**
  - We use the default value for regression trees (a third of the variables), but also experimented with other values, small differences in results
- **Number of trees**
  - 500 trees, selected by assessing "stabilization" of out-of-bag error in the first training sample

# Data: Targets

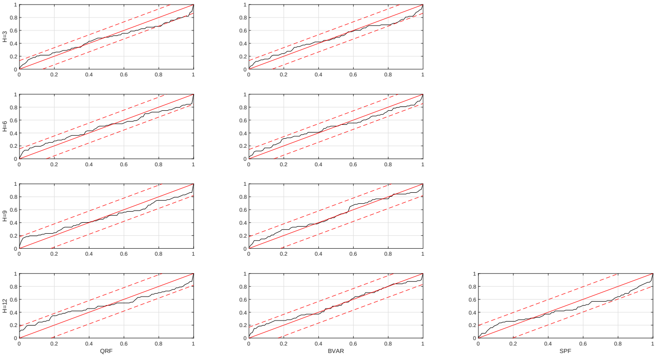
Figure: Headline and Core Inflation - year-on-year




Note: Headline inflation: black solid line; Core inflation: blue dashed line.

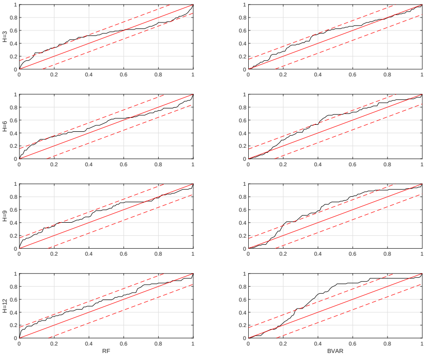
Sample: January 1992 - December 2022

# Calibration Headline Inflation - Rossi and Sekhposyan (2019)'s test of uniformity of PITs




Note: Red lines: 1% critical values of the Kolmogorov-Smirnov test of PIT uniformity (dashed) and 45% degree line; Black line: Cumulative distribution function (CDF) of the PITs.

# Calibration Core Inflation - Rossi and Sekhposyan (2019)'s test of uniformity of PITs




Note: Red lines: 1% critical values, Kolmogorov-Smirnov test of PIT uniformity (dashed) and 45% degree line; Black line: Cumulative distribution function (CDF) of the PITs.

# Out-of-sample accuracy - CRPS

| Horizon | QRF | BVAR |
|---|---|---|
| Panel a: Headline Inflation | | |
| **h=3** | 0.29 | 0.28 |
| **h=6** | 0.50 | 0.49 |
| **h=9** | 0.74 | 0.67 |
| **h=12** | 0.93 | 0.88 |
| Panel b: Core Inflation | | |
| **h=3** | 0.14 | 0.14 |
| **h=6** | 0.23 | 0.24 |
| **h=9** | 0.31 | 0.32 |
| **h=12** | 0.37 | 0.39 |

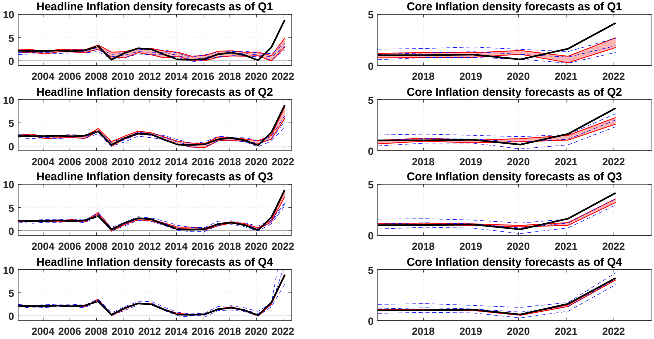
Note: CRPS for QRF (second column) and VARCOMB (third column)

# Out-of-sample accuracy - RMSE

| **Horizon** | **QRF** | **BMPE** |
|---|---|---|
| Panel a: Headline Inflation | | |
| **h=3** | 0.58 | 0.47 |
| **h=6** | 0.92 | 0.94 |
| **h=9** | 1.48 | 1.42 |
| **h=12** | 1.97 | 1.65 |
| Panel b: Core Inflation | | |
| **h=3** | 0.21 | 0.22 |
| **h=6** | 0.36 | 0.38 |
| **h=9** | 0.64 | 0.58 |
| **h=12** | 0.82 | 0.68 |

Note: RMSE for QRF (second column) and BMPE (third column)

# SPF - density forecast of QRF and SPF for current year




Note: Red Area: 16th to 84th quantile of the QRF, current year for headline inflation (left panels) and core inflation (right panels); Dashed Lines: 16th to 84th quantile of the SPF, current year for headline inflation (left panels) and core inflation (right panels).